NEW HORIZON COLLEGE
MARATHALLI, BANGALORE

1.5 QUANTITATIVE METHODS FOR BUSINESS - 1

I SEM BBA

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Number System

Natural Numbers:-

These are the numbers, which are used in counting the units of universe.

N= {1, 2, 3…}

Integers:-

A set of digits including zero having no fractional part is called an integers, or whole numbers. It is denoted by *Z* or *I*.

Z= {……. -3,-2,-1, 0, 1, 2, 3………}

Where -1,-2,-3… are called negative integers and 1, 2, 3…. are called positive integers.

Odd and Even Numbers:-

Odd numbers: - The numbers which don't have two as the factor. Eg: - 1, 3, 5, 7….. So on.

Even numbers: - The number having two as the factor. Eg: - 2, 4, 6 … so on.

Prime Numbers:-

The prime number is an integer other than (=0 and ±1) which is divisible by 1 and by itself only. Eg: - 2, 3, 5, 7…… are positive prime numbers.

Rational Numbers:-

A number of the form p/q where p and q are integers and (q not equal to zero) is called a rational number. Eg: - -6/7, 2/3……… so on.

Irrational numbers:-

A number which is not a rational is called irrational numbers. Eg: - √2, √3, 3√5….. So on.

Real Numbers:-

A set of all rational numbers together with sets of all irrational numbers is called real numbers. It is denoted by *R*. Eg: - √2, ½ …… so on.

Complex numbers:-

A number of the form a+ib where a and b are real numbers and i is an imaginary unit are called complex numbers. Eg: - 2+i3, -2-i4….. So on.

⇨ **Imaginary numbers:-**

If in a complex number the real part is zero then it is called imaginary number. Eg: - i3, -i5….. So on.

LCM (least common multiple):-

The least common multiple (L.C.M) of a given set of integers is the smallest integers which is divisible by each one of the given integers.

Find the LCM of 16, 24, & 36.

2	**16**	**2**	**24**	**2**	**36**
2	**8**	**2**	**12**	**2**	**18**
2	**4**	**2**	**6**	**2**	**9**
2	**2**	**3**	**3**	**3**	**3**
	1		**1**		**1**

HCF (highest common factor):-

The highest common factor (H.C.F) of two or more numbers is the highest number that divides each of them exactly and is the product of all their common prime factors.

Methods of finding H.C.F:-

Factor method

Find HCF of 16 & 24

2	**16**	**2**	**24**
2	**8**	**2**	**12**
2	**4**	**2**	**6**
2	**2**	**3**	**3**
	1		**1**

HCF = Product of least power of prime factor that appears.

16= 2x2x2x2x1 => 2

24= 2x2x2x3x1

Division method

find the HCF of 42 & 70

42) 70 (1

42

28) 42 (1

28

14) 28 (2

28

00

NOTE:-

The HCF and LCM of 2 integers are related by the following formulae:-

1. **LCM×HCF= product of 2 integers.**
2. **HCF= HCF of numerator/ LCM of denominator**
   LCM= LCM of numerator/ HCF of denominator

Find the HCF and LCM of 1/3, 5/6, 2/9 and 4/27?

Numerator**: 1,5,2,4**

Denominator: **3,6,9,27**

HCF of numerator:

The HCF of two numbers is 2 and LCM is 450. If one number is 50. Find the no.?

Sol: - LCM×HCF= product of 2 integers

Let the no. be X

450×2=50X

900=50X

X=900/5

X= 18

THEORY OF EQUATIONS

Polynomial Equations:- An Expression of the form $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots\ldots\ldots+a_n=0$ where $a\neq0$ and $a_0,a_1,\ldots\ldots.a_n$ are constant is called a polynomial equations of degree n in the variable x.

Eg:-

- ⇨ $2x+3=0$ is eq. of degree 1
- ⇨ $2x^2+4x+6=0$ is eq. of degree 2
- ⇨ $3x^3+2x^2+6x+9=0$ is eq. of degree 3 and so on.

Degree of Equations: - The highest power of the variable in an equation gives degree of an equation.

1ST degree linear equation:-

The equation of the form $ax+b=0$ (where $a\neq0$) is called a linear equation in one variable x.

2nd degree linear equation: - The equation of the form $ax+by=0$(where $a\neq0$ and $b\neq0$) is called a linear equation in 2 variable x and y.

Solve for x:-

1. **2x+4=0**
   - ⇨ $2x=-4$
   - ⇨ $x=-4/2$
   - ⇨ $x=-2$
2. **4(x+5)=6x+3**
   - ⇨ $4x+20=6x+3$
   - ⇨ $20-3=6x-4x$
   - ⇨ $17=2x$
   - ⇨ $X=17/2$

Quadratic Equations:- The Equation of the form $ax^2+bx+c=0$ where $a\neq0$ and a,b and c are constant is called a quadratic equation in one variable.

Eg: - $4x^2+3x+4=0$

The general form of quadratic equation is $\mathbf{ax^2+bx+c=0}$

Solve for x:-

1. $5(x^2+3)-12=3(x^2-9)+48$

⇨ $5x^2+15-12=3x^2-27+48$
⇨ $5x^2+3=3x^2+21$
⇨ $5x^2-3x^2=21-3$
⇨ $2x^2=18$
⇨ $X^2=18/2$
⇨ $X^2=9$
⇨ $X=\pm3$

Methods of solving quadratic equations:-

1. **Factorization Method**
2. **Formula Method**

Factorization Method:-

Solve for x:-

1. $\mathbf{X^2-4x+3=0}$

⇨ $x^2-3x-1x+3=0$
⇨ $x(x-3)-1(x-3)=0$
⇨ $(x-1)(x-3)=0$
⇨ $X=1$ and $x=3$

Problem:-

1. **Sum of 3 successive odd no. is 177. Find the no.?**
   Let the 3 odd no. be x.x+2, x+4

⇨ $X+x+2+x+4=177$
⇨ $3x+6=177$
⇨ $3x=177-6$
⇨ $3x=171$
⇨ $X=171/3$
⇨ $X=57$

By this we get:-

$X=57$

$X+2=57+2=59$

$X+4=57+4=61$

Simultaneous Equations: - A first degree equation in two variables, say 'x' and 'y', is called a linear equation in two variable. Two or more such linear equations in two variables 'x' and 'y' are called linear simultaneous equations or simple simultaneous equations.

Methods of solution:-

1. **Method of Elimination**
2. **Method of substitution**

1. **Method of Elimination**

1. x+y =2

x-y =4

⇨ X+y =2

x-y=4

2x=6

⇨ X=6/2

⇨ X=3

Put x=3 in eq. 1

3+y=2

⇨ Y= 2-3

⇨ Y=-1

If eqs. are added y gets eliminated and then the value of x can be easily found.

2. **Method of Substitution**

1. **X-y =6**

X+y =2

⇨ X-y =6 -----------1

⇨ X+y =2 -----------2

Consider eq. 1

X-y=6

X=6+y

Substitute x=6+y in eq. 2

X+y =2

⇨ 6+y+y=2

⇨ 6+2y=2

⇨ 2y=2-6

- ⇨ 2y=-4
- ⇨ Y= -4/2
- ⇨ Y= -2

Put y= -2 in eq. 1

- ⇨ x-(-2)=6
- ⇨ x+2=6
- ⇨ x=6-2
- ⇨ x=4

10 years ago the father was 4 times of his son. 10 years from now the age of the father will be double that of his son. What are the present age of father and son?

Sol: - let the father age be x

And let the son age be y

Before 10 years

Father -> x-10

Son-> y-10

- ⇨ X-10=4(y-10)
- ⇨ X-10=4y-40
- ⇨ X-4y= -30 -----------1

10 years from now

Father age-> x+10

Son age -> y+10

- ⇨ X+10=2(y+10)
- ⇨ X+10=2y+20
- ⇨ X-2y=10 -------------2

Using substitution method

X= -30+4y

- ⇨ -30+4y-2y=10
- ⇨ -30+2y=10
- ⇨ 2y= 40
- ⇨ Y=40/2

⇨ Y= 20

Put y=20 in eq. 1

⇨ x-4(20)= -30

⇨ X-80= -30

⇨ X= -30+80

⇨ X= 50

PROGRESSIONS

SEQUENCE REAL NO'S

Definition:- a **set of real no's written in succession according to some rules is said to form a sequence of real no's.the successive numbers in the sequence are called its terms or elements.**

a sequence is usually denoted by$\{x_n\}$,where

$\{xn\} = x_1, x_2, x_{3,\ldots\ldots}x_n$

the no's $x_{1,}x_{2,}x_3$,……..are called the elements of the sequence $\{x_n\}$ and x_n is called the n^{th} element of the sequence.

Note:-it is not always true that a sequence can be expressed by a simple algebraic expression involving n. for example, consider the sequences,

the finite series:-1,1⁄2,1⁄3,1⁄4,….1⁄20

2,4,6,8,10……………32

the infinite series:-$x_{1,}x_2,x_{3,\ldots}x_{n,\ldots\ldots}$

and is denoted by $\sum x_n$

if a series has finite terms then it is called finite series.

Arithmetic Progression

A sequence of no's in which different elements(except first)are written by increasing(or decreasing)its previous element

by the same quantity, is called an arithmetic progression(a.p).

Example: 7,10,13,………….

The general form of an a.p can be written as,

a,a+d,a+2d,a+3d,…….

thus the nth element of a.p with ‘a’ as first element and ‘d’ as common difference is a+(n-1)d.

Example1:-Find the n^{th} term and the 13^{th} term of the a.p.7,10,13,………….

solution:in the given a.p. a=7,d=10-7=13-10=3

now, n^{th}term=T_n=a+(n-1)d=7+(n-1)3=3n+4

now, T_{13}=3(13)+4=43

Example2:-Find the 3 no’s which are in a.p.whose sum is 12 and the sum of their cubes is 408.

solution: let the no’s be

a-d , a , a+d

these no’s are in a.p.whose common difference is d.

by data, a – d + a + a + d = 1 2 => 3 a = 12 => a=4

again by data

$(a-d)^3+a^3(a+d)^3=408 \quad \Rightarrow 3a^3+6ad^2=408$

⇨ $192+24d^2=408$

⇨ $24d^2=216 \quad \Rightarrow d^2 = 9 \Rightarrow d=\pm3$

if a=4,d=3,the required no’s are 1,4,7

if a=4,d=-3,the required no’s are 7,4,1

observe that we got the same no’s for both d=3 and d=-3

Sum to ‘n’ terms of a.p.

a , a+d , a+2d , +… … … … ...+ a(n-1)d

is given by,

$s_n=n/2[2a+(n-1)d]$ or

$s_n=n/2[a+l]$ where, l= T_n = a+(n-1)d

Note:

The first formula is used to find the sum of n terms of an a.p. if the first element and the common difference are known. Where as the second formula is used to find the sum to n terms ,if the first element and the n^{th} elements are known.

Example 1 :Find the sum of 30 elements of the a.p.1,3,5,7,……..

solution:here a=1 d=2 and n=30

now we have,

$s_n=n/2[2a+(n-1)d]=30/2[2+(30-1)2]=15\times60=900.$

Example 2 : The first term of an a.p. is 42 and the sum of the first five terms is 178.5.find the fifth term.

solution: $a=4.2$, $n=5$ and $s_5=178.5$

we have $s_5=5/2(a+l)$ ($l=5^{th}$ term)

⇨ $178.5=5/2(4.2+l) \Rightarrow 357=2l+5l \Rightarrow l=67.2$

that is the 5^{th} term=67.2

Geometric Progression

Definition:-A geometric progression is a sequence of no's in which the ratio of every element to its previous element is a fixed constant.

this fixed constant is called common ratio.

Example: 1,4,16,……..

The general form of a geometric sequence can be written as

$$a, ar, ar^2, ar^3, \ldots\ldots ar^{n-1}, \ldots..$$

here a is called the first element and r is called the common ratio of the g.p.

$$ar/r = ar^2/ar = ar^3/ar^2 = \ldots\ldots\ldots\ldots = ar^{n-1}/ar^{n-2} = \ldots\ldots\ldots\ldots = r$$

the element ar^{n-1} is called the nth element or the general term of the g.p. and it is denoted by T_n.

i.e. $T_n=ar^{n-1}$

Example1: Find the 7th and 9th elements

$$4, 1, 1/4, 1/16, \ldots\ldots..$$

solution:in the given g.p. $a=4$, $r=1/4$

now, $T_7=a.r^{7-1}=4.(1/4)^6=1/4^5$

$T_9=a.r^{9-1}=4.(1/4)^8=1/4^7$

sum to n terms of g.p.

$$s_n=a(r^{n}-1)/r-1$$

thus, if a is the first element and r is the common ratio of the g.p.,then the sum of n elements is as given above.

example1: find the sum of 4 elements of the g.p. 1,4,16,……..

solution:here $a=1$, $r=4$.

now, $s_n=a(r^n-1)/r-1$

$$s_n=1(4^4-1)/4-1=255/3=85.$$

i.e., sum of 4 terms, $1+4+16+64=85$

Arithmetic Mean and Geometric Mean

Arithmetic mean

Definition: If three quantities a,a,b are in a.p., then a is called the arithmetic mean of a and b.

thus $a=a+b/2$

example: Insert 8 arithmetic mean between 2 and 11.

solution: let the required a.m.'s be $a_1, a_2, a_3, \ldots\ldots a_8$ by definition

$2, a_1, a_2, a_3, \ldots\ldots a_8, a_{11}$ are in a.p. with a=2.

now, $11 = 10^{th}$ element => 2+9d=11 => d=1

thus the common difference is d=1.hence the means are

3,4,5,6,7,8,9,10.

Geometrical mean

Definition: If three quantities a,g,b are in g.p., then g is called the geometric mean(g.m.)of a and b.

thus $g/a = b/g \Rightarrow g^2 = ab \Rightarrow g = \sqrt{ab}$

MATRICES AND DETERMINANTS

Matrices: - An arrangement of numbers (real or complex) in the form of rows and columns within a bracket is called Matrices.

Examples:-

1. $\begin{pmatrix} 2 & -1 \\ 3 & 2 \\ 4 & 2 \end{pmatrix}$ is a matrix of order 3x2

2. $(i\ 2i\ 0)_{1x3}$ is a matrix of order 1x3

Types of matrices:-

1. **Null matrix or zero matrix:-**
   A matrix in which each element is a zero is called a null matrix or zero matrixes and denoted by *O.*
   Eg: - (0 0 0) is a zero matrix of order 1x3.
2. **Row matrix:-**
   A matrix having only one row is called a row matrix. That is a matrix of *1xn* is a row matrix.
   Eg: - $(1\ 3)_{1x2}$
3. **Column matrix:-**
   A matrix having only one column is called a column matrix. That is a matrix of *mxl* is a column matrix.
   Eg: $\begin{pmatrix} -1 \\ 2 \end{pmatrix}_{2x1}$
4. **Square matrix:-**

A matrix in which the number of rows equal to number of columns is called a square matrix.

Eg: $\begin{bmatrix} -2 & 1 \\ -1 & 3 \end{bmatrix}$

5. Rectangular matrix:-

A matrix in which the number of rows is not equal to the number of columns is called rectangular matrix.

Eg:- $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}_{2x3}$

6. Diagonal matrix:-

A square matrix, in which all the elements except the diagonal elements are zero, is called diagonal matrix.

Eg:- $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}_{2x2}$

7. Scalar matrix:-

A diagonal matrix in which all the diagonal elements are equal is called a scalar matrix.

Eg:- $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}_{2x2}$

8. Unit matrix or Identity matrix:-

A diagonal matrix, in which each diagonal entry is unity, is called unity matrix or identity matrix.

The unit matrix of order n is denoted by I_n.

Eg: - $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2x2}$

Algebra of matrices:-

1. **Equality of two matrices:**
   Two matrices are said to be equal, if their orders are same and the corresponding elements are equal.

2. **Addition of two matrices:-**

   Let A and B be two matrices of order $m\times n$. The sum $A+B$ of the matrices A and B is a matrix of order $m\times n$ and whose elements are the sum of the corresponding elements of A and B.

3. **Scalar multiple of a matrix:-**

   Let A be a matrix of order $m\times n$ and k be any scalar. The scalar multiple $k\cdot A$ of the matrix A with scalar k is a matrix of order $m\times n$ and its elements are obtained by multiplying each element of a by k.

NOTE:-

IF A, B and C re matrices of same order, say m×n, then

(a) ***$A+B=B+A$***

(b) ***$A+(B+C)=(A+B)+C$***

(c) ***$k\cdot(A+B)=k\cdot A+k\cdot B$ where k is a scalar***

(d) ***$A+O=O+A=A$ where O is the zero matrix of order m×n.***

(e) ***$A+(-A)=-A+A=O$ where –A is the matrix obtained from A by multiplying each element of A by -1. The matrix –A is called the negative of the matrix A.***

Problem:-

If $A=\begin{pmatrix} 2 & 3 & 4 \\ -3 & 1 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 3 & -4 & -5 \\ 1 & 2 & 1 \end{pmatrix}$ and $C=\begin{pmatrix} 5 & -1 & 2 \\ 7 & 0 & 3 \end{pmatrix}$. Find the matrix X such tat $2A+3B-X=C$?

Sol: - we have,

$$2A+3B-X=C \quad \Rightarrow X=2A+3B-C$$

Consider, $2A+3B=\begin{pmatrix} 4 & 6 & 8 \\ -6 & 0 & 4 \end{pmatrix}+\begin{pmatrix} 9 & -12 & -15 \\ 3 & 6 & 3 \end{pmatrix}$

$\Rightarrow 2A+3B=\begin{pmatrix} 13 & -6 & -7 \\ -3 & 6 & 7 \end{pmatrix}$

Now, $2A+3B-C=\begin{pmatrix} 13 & -6 & -7 \\ -3 & 6 & 7 \end{pmatrix}-\begin{pmatrix} 5 & -1 & 2 \\ 7 & 0 & 3 \end{pmatrix}$

$X=\begin{pmatrix} 8 & -5 & -9 \\ & & \end{pmatrix}$

-10 6 4

4. Multiplicative of matrices:-

Consider two matrices *A* and *B* where *A* is of order $m\times n$ and *B* is of order $n\times p$. Here, the number of columns of *A* is equal to the number of rows of *B*. Two such matrices are said to be **compatible**.

NOTE:-

(a) ***If A,B and C are three matrices whose order respectively are $m\times n, n\times p$ and $p\times n$ then $A\cdot(B\cdot C)=(A\cdot B)\cdot C$***

(b) ***If A is a matrix of order $m\times n$ and B,C are matrices of order $n\times p$, then $A\cdot(B+C)= AB+AC$***

(c) ***If A is $m\times n$ matrix, $A\cdot I_n= A$ and $I_m\cdot A=A$.***

5. Transpose of a matrix:-

Let *A* be a matrix of order $m\times n$. The matrix obtained from *A* by interchanging the rows into columns is called the transpose of the matrix *A* and it is denoted by *A'* or A^T.

Let A and B be any two matrices of same order, then

(a) ***(A+B)'= A'+B***

(b) ***(A')' = A***

(c) ***If A and B are two matrices such that A·B is defined then (A·B)'= B'·A'***

If $A=\begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \end{pmatrix}_{2x3}$ then $A'=\begin{pmatrix} 2 & 1 \\ -1 & 2 \\ 3 & 4 \end{pmatrix}_{3x2}$

Problem:-

If $A=\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ verify (AB)'= B'A'

Sol: - $AB=\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

⇨ $\begin{pmatrix} 6+1 & 2-2 \\ 9-2 & 3+4 \end{pmatrix}$ => $\begin{pmatrix} 7 & 0 \\ 7 & 7 \end{pmatrix}$

(AB)'= $\begin{pmatrix} 7 & 7 \\ 0 & 7 \end{pmatrix}$

$B'A'=\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$ => $\begin{pmatrix} 6+1 & 9-2 \\ 2-2 & 3+4 \end{pmatrix}$

⇨ $\begin{pmatrix} 7 & 7 \\ 0 & 7 \end{pmatrix}$

⇨ (AB)' = B'A'

DETERMINANTS

Determinants:-

To every square matrix *A* we associate a unique number called the determinant of the matrix *A*. The determinant of the square matrix *A* is denoted by *IAI.*

If A is an $n \times n$ square matrix then *IAI* is called the determinant of order n or n^{th} order determinant. In this section we restrict our self to the second order and third order determinants.

Expansion or Evaluation of a determinant:-

(a) Determinant of a second order

Let A= $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ be any 2x2 matrix. Then we define

The determinant of A, denoted by determinant A or *IAI* as

$$IAI = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \quad \text{(replacing the bracket by vertical lines)}$$

$$= a_1b_2 - a_2b_1$$

Observe that product remains unchanged with downwards arrow while it is changed with upward arrow.

The number $a_1b_2-a_2b_1$ is called the value or the expansion of the determinant of *A* i.e.., the value of $|A|$.

(b) Determinant of third order

Consider a 3×3 square matrix

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

Then the determinant of *A* is denoted as

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

This is a determinant of third order. We expand or find its value with the help of the elements of any row or column. We shall see how we define the expansion of $|A|$ with the help of first row.

Multiply each element of the first row by the corresponding determinant of second order obtained by deleting the row and column in which the element is lying and add these products by taking the signs before the products as alternatively positive and negatives. Thus,

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

⇨ $a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

⇨ $\mathbf{a_1(b_2c_3-c_2b_3)-b_1(a_2c_3-c_2a_3)+c_1(a_2b_3-b_2a_3)}$

Example:-

1. Find 'x' if

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 3 \\ 3 & 4 & 3 \end{vmatrix} = 0$$

Sol: $\begin{vmatrix} -1 & 2 & 3 \\ 2 & x & 3 \\ 3 & 4 & 3 \end{vmatrix} = 0$

⇨ $1(3x-12)-2(6-9)+3(8-3x)=0$

⇨ $3x-12+6+24-9x=0$

⇨ $-6x+18=0$

⇨ $6x=18$

⇨ $X=18/6$

⇨ $X=3$

Solution of linear equations- Cramer's Rule

Consider the system of equations

$$a_1 x+b_1 y=c_1$$

$$a_2 x+b_2 y=c_2 \text{ ------------------ (1)}$$

Let $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, Δ is the det. Formed by taking the coefficients.

Consider, $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

⇨ $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

⇨ $\Delta_1 = \begin{vmatrix} c_1 & b_1 \end{vmatrix}$

$c_2 \; b_2$

⇨ $\Delta_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

This method of solving the system of equations is called **Cramer's rule**.

Minors and Cofactors:-

Consider a matrix A of order 3×3

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Minor of an element: -

The minor of an element a_{ij} is the determinant of the submatrix obtained by deleting the i^{th} row and j^{th} column of the matrix A. The minor of a_{ij} is denoted by M_{ij}.

In the matrix A, above

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{23}a_{32}$$

$$M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = a_{11}a_{23} - a_{13}a_{21}$$

Cofactor of an element:-

The cofactor of an element a_{ij} of a matrix A, is denoted by A_{ij} and it is defined as

$$A_{ij} = (-1)^{i+j} M_{ij}$$

Inverse of a matrix

Let A be a square matrix of order n×n .If there exists a matrix B of order n×n such that AB=BA=In , then B is called inverse of the matrix A.

$A^{-1} = 1 \backslash |A| . (\text{adj } A)$

Find the inverse of the matrix $A = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$

solution: the inverse of a non singular matrix A is given by

$A^{-1} = \frac{1}{|A|} \text{ adj } A$

Example1: First we shall find the cofactors of the elements of A

Solution: A11 =+(-2), A21=-(-1)=1, A22=+2

Adj a=$\begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix}$ and |A| =-4+3=-1≠0

$A^{-1} = \frac{1}{|A|}$ adj A $\Rightarrow$ $A^{-1} = \frac{1}{-1}\begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$

Example2: find the inverse of the matrix $\begin{pmatrix} 2 & -1 & 3 \\ -1 & 4 & 2 \\ 0 & -3 & 1 \end{pmatrix}$

Let A=$\begin{pmatrix} 2 & -1 & 3 \\ -1 & 4 & 2 \\ 0 & -3 & 1 \end{pmatrix}$

The cofactors of thr elememts are given below.

A_{11}=cofactor of 2 = +$\begin{vmatrix} 4 & 2 \\ -3 & 1 \end{vmatrix}$=4+6=10

A_{12}=cofactor of -1= -$\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix}$=-(-1-0)=1

A_{13}=cofactor of 3= +$\begin{vmatrix} -1 & 4 \\ 0 & -3 \end{vmatrix}$=3-0=3

A_{21}=cofactor of -1= -$\begin{vmatrix} -1 & 3 \\ -3 & 1 \end{vmatrix}$= -(-1+9)= -8

A_{22}=cofactor of 4=+$\begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix}$= 2-0=2

A_{23}=cofactor of 2= -$\begin{vmatrix} 2 & -1 \\ 0 & 3 \end{vmatrix}$= -(-6-0)=6

A_{31}=cofactor of 0=+$\begin{vmatrix} -1 & 3 \\ 4 & 2 \end{vmatrix}$= -2-12= -14

A_{32}=cofactor of -3= -$\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix}$= -(4+3)=-7

A_{33}=cofactor of 1= +$\begin{vmatrix} 2 & -1 \\ -1 & 4 \end{vmatrix}$= 8-1 =7

|A|= $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

=2(10)+(-1)(1)+3(3) =20-1+9 =28≠0

Now, $A^{-1} = \frac{1}{|A|}$adj A $= \frac{1}{|A|} . \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$

$$=\frac{1}{28}\begin{pmatrix}10 & -8 & -14\\ 1 & 2 & -7\\ 3 & 6 & 7\end{pmatrix}=\begin{pmatrix}\frac{5}{14} & \frac{-2}{7} & \frac{-1}{2}\\ \frac{1}{28} & \frac{1}{14} & \frac{-1}{4}\\ \frac{3}{28} & \frac{3}{14} & \frac{1}{4}\end{pmatrix}$$

Commercial Arithmetic

Definition of Interest :

Interest is the compensation received by the lender of money from the borrower, calculated at a specified rate percent and for a specified time, on the sum of money lent by the lender to the borrower.

Symbols used in simple and Compound Interest

p=principal

n=number of periods of which the money is lent

r=rate of interest present per annum, expressed as a decimal

I=rupees of Interest earned for the interest period

F=amount to be received at the end of 'n' periods. Future Value

Types of Interest

If an amount of money is lent, interest accrues at regular time intervals. Each time interval constitutes an interest period. the interest earned on the original amount is calculated according to a specified rate of interest at the end of each interest period. To calculate the interest, two different methods can be used "simple interest" or "compound interest"

Simple Interest

The 'simple interest' approach considers that the interest earned is a linear function of time, with the original sum lent and the rate of interest remaining constant. In the other words, simple interest earned directly varies with the 'time'

F=p+I

I = F - p

I = p r n

F - p = p r n

F = p + p r n

F = P [1 + r n]

year	Principal at the beginning of the year	r = 5%	Simple interest	F= Future Value
1	Rs.100	0.05	5.00	100
2	Rs.100	0.05	5.00	100
3	Rs.100	0.05	5.00	100
4	Rs.100	0.05	5.00	100
5	Rs.100	0.05	5.00	100
	Total interest=25			

Calculate the simple interest on 500 for 5 years at 6% per annum .

We know that I = P r n

I=simple interest

P=the principle amount lent i.e 500 Rs

r=rate of interest i.e 6% per annum(0.06)

n=the time period i.e 5 years

I=500*0.06*5=150

Exact and ordinary simple interest

Exact simple interest it is calculated on the basis of a 365 day year (366 days in case of a leap year).

Ordinary simple interest is calculated on the basis of a 360 day year. While the exact simple interest is used in commercial practice. The ordinary simple interest is used mostly by banks. Use of ordinary simple interest magnifies the interest charged by the lender.

Exact and approximate time

When dates are given, the number of days for which interest is to be computed is determined in two ways.

1. Exact time, as the name implies , is the actual number of days as found from a calendar.
2. Approximate time is found by assuming each month to have only 30 dyas.

Example: find the exact and the approximate time from 20^{th} June 2003 to 24^{th} august 2003.

Solution:

Exact time-

No, of days= number of days remaining in June + no, of days in July + no. of days in august till the date indicated.

= 11 days in June (indicate date inclusive) + 31 days in July + 23 days in August (indicate date exclusive)

=65 days

Approximate time-

We take,

August 24 , 2003 as	24-8-2003
June 20 , 2003 as	20-6-2003
Subtracting we get	04-2-0

That is , 4 days and 2 months.

Taking 30 days in a month , this will add up to 2*30+4=64 days.

COMPOUND INTEREST

The compound approach assumes that the interest earned is not withdrawn at the end of the interest period but is added, automatically, to the original sum lent to constitute the principal for the second interest period. This process is repeated for the entire time period 'n'. the total interest thus accumulated is called the "compound interest".

Formula:

$$F_n = p(1+r)^n$$

C. $I = p[(1+r)^n - 1]$

year	Principal at the beginning of the year	R =5%	Compound Interest earned	F_n = Future value
1	Rs.100	0.05	5.00	105.00
2	Rs.105	0.05	5.25	110.25
3	Rs.110.25	0.05	5.51	115.76
4	Rs.115.76	0.05	5.79	121.55
5	Rs.121.55	0.05	6.08	127.63
	Total interest=27.63			

Important formulae in compound interest

1. To find the amount due after 'n' years where 'n' is a positive number.
   F=P$(1+r)^n$
2. To find the present value (or, the principal) when F_n ,r and 'n' are given.
   P=$\frac{F_n}{(1+r)^n}$

Example1. Calculate the amount and compound interest on Rs.100 for 15 years, allowing compound interest at the rate of 12% per annum.

Solution:

Given:

P=Rs.100

N=15 years

R=12%=12/100=0.12

Required to find F_n and I

To find F_n:

F=P$(1+r)^n$

F=100(1+0.12)15=1oo$(1.12)^{15}$

=Rs.547.00

To find I:

I=F-P

=Rs.547.00-Rs.100.00

=Rs.447.00

Nominal and effective rates of interest

$$R=[1+\frac{R}{q}]^{q} -1$$

Where

R=rate of interest(i.e., contracted or the agreed rate of interest)

r= effective rate of interest

q= the number of times interest is computed in a year i.e., the number of conversion periods in a year.

Example1: find the nominal and effective rates of interest in each of the following case:

(1) Rs.1000 lent at 12% p.a, interest payable half yearly

Solution: to compute nominal and effective rates of interest we need to know only the given rate of interest and the number of times interest is payable or computed or reckoned in a year. All other information is not required. Using the formula in 7.5, we get

(1) Nominal rate is the given rate per annum=12%
Given R =12%
Q=2 (i.e., number of times interest compound in a year)
Required to find the effective rate'r'

$$r=\left[1+\frac{R}{q}\right]^{q} -1$$

$$=[1+\frac{0.12}{2}]^{2} -1$$

$$=(1.06)^{2} -1$$

=1.1236-1=0.1236=12.36%

Annuities

Definition: Fixed sum of money payable periodically at equal intervals of time under certain conditions.

Examples: Salary, interest, pension and rent etc.

Annuities may be payable daily or weekly or monthly or quarterly or half yearly or annually

Definition: An annuity is a fixed sum of money payable periodically at equal intervals of time under certain conditions.

e.g salaries, interest , pension ,LIC premier ,rent etc

Annuities may be payable daily or weekly or monthly or quarterly or half yearly or annually……….

Annuities may be broadly classified as follows:

01.**Annuity certain**: an unconditional agreement to make a fixed(FINITE) number of annuity payments, either at the beginning or at the end of the period, is called as annuity certain.

02**.Annuity contigent**: if the payment of an annuity or the duration of annuity payments or both are subject to the happening of any event. the annuity is called annuity contingent.

03**.Perpetuity**: an annuity which is for an infinite period is called perpetuity.

04.**Deferred annuity**: when the annuity payments begin only after the lapse of certain specified period of time, the annuity is called deferred annuity.

An annuity certain can be further classified as:

(a) Annuity immediate: When the annuity payments are payable at the end of each stipulated time period, called as annuity immediate.
(b) Annuity due: when the annuity payments are payable at the beginning of each stipulated time period, called as annuity due.

Annuities in Economics and Business

(a) Buying of durable consumer goods under hire purchase/ installment scheme-the equated monthly installments are annuities.
(b) Bank loans are taken for various purposes and loans are repayable in installments which may be annuities'
(c) We can invest fixed interest bearing securities like debentures ,government bonds.etc and interest receivable on them, form annuities.
(d) Businessmen's investment money is profitable projects and the cash flows there from can be I the

form of annuities.

(e) Assets like building, land, etc can be leased and the lease rentals are annuities.
(f) The premier payable on life insurance policies are in the form of annuities.
(g) The monthly deposit we make on life insurance policies are in the form of annuities.

Future value of annuity immediate

Before we go to the application of the formula, please remember that annuity immediate is payable at the end of each time period. Each time period may be a day or a week or a month or a year, etc., generally unless otherwise stated, it is assumed to be a year.

$$F= \frac{A[(1+r)^n -1}{r}$$

Example1: calculate the amount of an annuity of Rs.5000 for 15 years, if the rate of interest is 12% p.a.

Solution: nothing is mentioned about the type of annuity. Therefore, it should be taken as annuity immediate.

Given: annuity (A)= Rs.5000; n=15 years and r=0.12

$$F= \frac{A[(1+r)^n -1}{r}$$

$$=\frac{5000[(1.12)^{15} -1]}{0.15}$$

$$=\frac{5000[5.4702-1]}{0.15}$$

$(1.12)^{15}$=A.I(15log1.2)

$$=\frac{5000*4.4702}{0.15}$$

=A.I(15*0.0492)

= Rs.1,49,006.97

=A.I(0.7380)

=5.4702

Future value of annuity due

Remember that annuity due is payable at the beginning of each period where as annuity immediate is payable at the end of each time period.

$$F=\frac{A[(1+r)^n -1}{r}(1+r)$$

Example1. Suppose you open a recurring deposit A/C with a company and put in Rs.1000 at the beginning of each year for 5 years. The company agrees to pay interest af 15 % p.a. how much will you get at the end of 5 years?

Solution: payments will be made at the beginning of each year. Therefore it is an annuity due.

F=$\frac{A[(1+r)^n-1]}{r}(1+r)$

=$\frac{1000[(1+0.15)^5]}{0.15}(1+0.15)$

=$\frac{1000[(1.15)^5-1]}{0.15}(1.15)$

=$\frac{1000(1.1631)}{0.15}$

=Rs.7753.75

Perpetuity

Future value of a perpetuity does not exist(it ia an infinite value). However, the present value of a perpetuity can be found by the following formula.

$$P_\infty = \frac{A}{r}$$

p_∞ is the present value of perpetuity.

Example1. What is the present value of an income of Rs.2000 a year to be received for ever? Assume the discount rate to be 16%.

Solution: the income is to be received for ever. Therefore, it is the perpetuity. The present value of a perpertuity is

$$P_\infty = \frac{A}{r} = \frac{2000}{0.16} = Rs.12{,}500$$

BILLS OF EXCHANGE

A bill of exchange (or hundi) is a negotiable instrument by which one person (the drawee of the acceptor) undergoes to pay another person (the drawer of banker) a certain sum of money at the specified date.

To understand the concept and the procedure of the bill discounting, it is necessary to know the basic form of a bill of exchange and the different terms associated with it.

A bill of exchange, generally looks like a fig.1

1.a B/E is always on the stamped paper.

2.drawer is T.V.N Rao

Payee is T.V.N Rao or his order(i.e any other person ordered by him)

Drawee is G.K.Ranganath

Acceptor is G.K.Ranganath

STAMP 163,BRIDGE ROAD

BANGLORE

DATED:15-05-2004

To,

Mr.G.K.Ranganath

Jayanagar banglore-11

Three months after date, pay to me on my order the sum of Rs 1000(Rupees one thousand only) for value received.

Rs 1000 accepted.

(sd) (sd)

G.K.Rangnath T.V.N Rao

Banker's discount

It is the simple interest calculated by the banker on the "face value" of the bill(F). the banker's discount(BD) is calculated for the discount period, using the discount rate.

$$B.D = F.t.r \qquad \text{……..(1)}$$

Where F=face value

t=discount period (time)

r= rate of interest (discount rat e

true discount (td)

true discount is the difference between the face value of the bill and the truth present worth. It is the interest calculated on the true present value. It is different from banker's discount. While BD is charged on the "face value", TD is calculated on the "true present worth". BD is always greater than TD. The difference between BD and TD is called the "banker's gain".

Present worth(present value):
Present value or present worth of two types.

(a) Banker's present value
(b) True present value.

(a) Banker's present worth: also known as the "discounted value of the bill", it is the difference between the face value and discount charged and deducted by the banker. In bill discounting transaction the banker will deduct the discount amount from the **face value** of the bill and pay only the net amount to the trader.
Bankers present value or discounted value of the bill = F-BD
Putting BD=Ftr from (1)
We get bankers present value= F-Ftr
= F(1-tr)

(b) True present value: we have seen that bankers charge interest on the face value to calculate banker's discount, but interest should normally be charged on the principal. The principal amount on which interest should have been computed is the true present worth. Because of banker's practice of charging discount on the face value, the banker present value will be lower than the true present worth. This is due to the banker's discount being higher than the true discount.
True present worth(p) =$\frac{F}{1+tr}$

Banker's gain(BG):
As stated earlier banker's gain is the difference betweenBD and TD.
BG=BD-TD
It is also equal to interest on TD
BG=TD.t.r

Example: a bill for Rs.1712.75 was drawn on 3-4-96 and made payable 3 months after date. It was discounted on 15-4-96 at 16.5% p.a. what was the discounted value of the bill and how much has the banker gained on this transaction.

Solution: in this problem date of drawing, bill period, date of discounting have been given. We have to determine the legally due date and discount period.

Legally due date= date of drawing + bill period 3 months + grace period of 3 days

= 3-4-96

=+0-3-0

=+3-0-0

Leaglly due date=6-7-96

Discount period= no. of days from date of discounting to legally due date.

Discounted on 15 april 1996

(15.4.96 to 30.4.96)	15 days
May 1996	31 days
June 1996	30 days
Due on 6th july 1996	06 days
Total	82 days

(a) Discounted value of bill= F(1-tr)

$=1713.75(1-\frac{82}{365}*0.165)$

$=1713.75(0.9629)=$ Rs.1650.17

(b) Banker's gain = BD-TD

(1) BD=Ftr

$=1713.75 * \frac{82}{365} * 0.165=$ Rs.63.53

(2) $P=\frac{F}{1+tr}$

$=\frac{1713.75}{1+\frac{82}{365}*0.165}=$Rs.1652.50

(3) TD= ptr

$=1652.50*\frac{82}{365}*0.165=$Rs.61.26

Therefore BG=BD-TD

=63.53-61.26= Rs.2.27

Ratio

Definition: a ratio is the relationship between two quantities of the same kind, with respect to their magnitude and denotes how many times one of the quantities is contained in the other.

Expression of a ratio

If there are two quantities 'a' and 'b', the relationship between them can be expressed as a ratio a:b, read as "a is to b".

The ratio a:b can also be expressed as a fraction a/b where in the relationship between 'a' and 'b' is expressed as "how many times 'a' is to 'b'".

Antecedent and consequent

In the ratio a:b, the first term 'a' is called the antecedent and the second term 'b' is called consequent.

Example of a ratio: suppose mr.X is getting a salary of Rs.15,000 p.m and Mr.Y is getting Rs. 5,000 p.m, the ratio of their salaries is Rs. 15,000 : Rs. 5,000. Thus the salary of Mr. X is thrice that of Mr. Y; because RS.15,000: Rs.5000 can also be written as $\frac{15000}{5000} = \frac{3}{1}$ or simply 3:1

Rules relating to ratios

1. A ratio can exist only between quantities of the same kind. For example, we can compare the salaries of two or more persons because salaries are expressed in the same currency. We cannot, however, compare 4kgs with 30 cows. Therefore, we cannot express as a ratio what we cannot compare.
2. When the two terms of ratio are interchanged, we get a second ratio which is called 'the inverse ratio" of the first. For example, for a given ratio a:b, b:a is called the inverse ratio of a:b. inverse ratios are also known as "reciprocal ratios".
3. A ratio is simply the result of comparison of two or more quantities or numbers. It is an abstract number which is quite distinct from the original magnitudes from which the comparisons were made. Or example, a particular household consumes in a month 20 kgs of rice and 5 kgs of wheat. thus, when we compare the consumption of rice and wheat in he form of a ratio, we would write 20 kgs: 5 kgs which is equal to 4:1. Hence , the ratios is 4:1 which are abstract numbers and are nit affected by the original measures of magnitude , i.e., kilograms. The ratio 4:1 merely suggests that the consumption of rice is four times the consumption of wheat. Further, the ratio should always be written in its lowest terms.
4. When two magnitudes of the same kind are being compared, they must be expressed in erms of a common unit of measurement before deriving a ratio. For example, Mr.X completes a job in 2 hours and Mr.Y completes the same job 40 minutes. Compare their performance. Here, for comparative purposes, the time should be expressed in similar terms. Accordingly, we can say Mr.X has taken 120 minutes(i.e., 2 hours) to 40 minutes. Taken by Mr.Y. thus, the ratio of time taken will be 120:40=3:1.
5. A ratio can also br expressed as a fraction. Therefore, the rules relating to fractions are equally applicable to ratios. Accordingly, if both the numerator and the denominator of a fraction are multiplied by the same quantity, the value of the fraction is not changed. Similarly, the ratio does

not get affected in value if both the terms are multiplied or divided by the same number.

6. When a certain quantity M is to be divide in a given ratio a:b, the two parts are

   $aM/a+b$ and $bM/a+b$

similarly, when a given quantity is to be divided in a given ratio a:b:c, the three parts are:

$aM/a+b+c$; $bM/a+b+c$ and $cM/a+b+c$

and so on.

For example, if Rs.1200 is to be divided amongst three persons, A ,B and C in the ratio of 3:4:5, each person will get.

A gets: $3*1200/3+4+5 =3*1200/12=$Rs.300

B gets: $4*1200/3+4+5=4*1200/12=$Rs.400

C gets:$5*1200/3+4+5=5*1200/12=$Rs.500

7. When the antecedents of a number of ratios are multiplied and the product denotes the antecedent of another ratio and the consequents are multiplied and the product denotes the consequent of the same ratio, the new ratio so obtained is called the “compounded ratio”.for example, if a:b; c:d; e:f; etc…. are a given set of ratios, the ratio obtained by compounding of all the ratios will be
   $a/b * c/d*e/f*\ldots\ldots\ldots=ace/bdf$
8. When the terms of a given raio are squared, the new ratio so obtained is called”duplicate ratio” of the given ratio.

Proportions:

When a ratio a:b is equal to another ratio c:d, the four terms a,b,c,d are said to be in proportions. Symbolically, the proportions is indicated as follows.

a:b=c:d

a:b::c:d

where :: is read “as to”.

The first and las terms of the proportion are called the ‘extremes’ and the second and third are called the ‘means’.

If a/b=c/d $\Rightarrow a, d \ are \ extremes$ and b,c are means.

Continued proportion:

If there are three or more terms a,b,c,d,……… which are so related that

a:b =b:c =c:d……

a/b =b/c =c/d =…………..

i.e.,

the terms a,b,c,d,……. Are said to be in ‘continued proportion’.

If a:b = b:c.

‘a’ is called the first proportional

‘b’ is called the mean proportional

‘c’ is called the third proportional.

Direct proportion

When two quantities of different kinds are so proportion are so related to each other that when one quantity increases or decreases in the same ratio, the ratio is said to be ‘varying directly’ and the proportion is called “direct proportional”.

If a:b = c:d

Ad = bc

Inverse proportion

When the terms of two magnitudes in a proportion are so related to each other that when one quantity decreases and when one quantity decreases, the other quantity increases, their ratios are said to be inversely proportional or vary inversely.

Compound proportion

A proportion that relates more than 2 ratios is called a compound proportion. If $a_1 : a_2$ is a ratio which is directly proportional to $b_1 : b_2$ and $c_1 : c_2$ in such a way that $a_1 b_1 c_1 = a_2 b_2 c_2 \ then \ a_1 : a_2; \ b_1 : b_2 \ and \ c_1 : c_2$ are said to be compound proportions.